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# ONE SCHEME OF ELECTROCHEMICAL MACHINING OF METALS BY A CURVILINEAR ELECTRODE TOOL

**N. M. Minazetdinov**

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*The nonlinear plane problem of the evolution of the shape of the metal surface (anode) during electrochemical machining by a curvilinear cathode of symmetric shape is solved. A condition is obtained which allows one to determine the position of the point of transition from the zone of anodic metal dissolution to the region in which machining stops.*

**Key words:** *electrochemical metal machining, ideal process, hydrodynamic analogy.*

**Introduction.** Improving existing methods and designing new computation techniques for electrochemical forming are important problems of the theory of electrochemical metal machining. A review of such problems and methods of their solution for an ideal process model is given in [1, 2].

In the present paper, a numerical-analytical solution of the two-dimensional problem of the evolution of the shape of the stationary anode boundary during metal surface machining by a curvilinear cathode is obtained for an ideal process model using methods designed to solve problems of jet flow over curvilinear bodies [3–5] and problems of the theory of filtration and explosion [6, 7].

**Process Model.** In the numerical-analytical solution of the problem, the model of the process described in [8] is used. We introduce a Cartesian coordinate system $(x_1, y_1)$ attached to the cathode which moves along the ordinate, and the complex potential of the electrostatic field $f(z_1) = v(x_1, y_1) + iu(x_1, y_1)$, where $z_1 = x_1 + iy_1$, $u(x_1, y_1)$ is the field potential, and $v(x_1, y_1)$ is the stream function. The potentials $u_a$ and $u_c$ on the surfaces of the anode and cathode are constant.

For the electrolytes which are solutions of sodium nitrate and chlorate, the dependence of the portion of the charge $\eta$ expended for metal dissolution on the anode current density $i_a$ can be written as [8]

$$\eta(i_a) = \begin{cases} 0, & i_a \leqslant i_{\rm cr}, \\ a_0 + a_1/i_a, & i_a > i_{\rm cr}, \end{cases}$$

where $a_0$, $a_1$, and $i_{\rm cr}$ are constants.

On the stationary anode boundary, the following condition is satisfied [8]:

$$\varkappa \frac{\partial u}{\partial n_a} = \frac{-a_1 + \rho V_c \varepsilon^{-1} \cos\theta}{a_0},$$

where $\varkappa$ is the specific electrical conductivity of the medium, $\varepsilon$ is the electrochemical equivalent of the metal, $\rho$ is the anode material density, and $\theta$ is the angle between the velocity vector $\boldsymbol{V}_c$ of motion of the cathode and the outward normal vector $\boldsymbol{n}_a$ to the given point of the anode boundary.

Kama State Academy of Engineering and Management, Naberezhnye Chelny 423810; nminazetdinov@yandex.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 51, No. 2, pp. 170–175, March–April, 2010. Original article submitted January 12, 2009; revision submitted May 27, 2009.

 

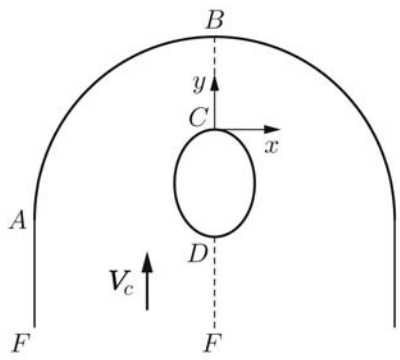


Fig. 1

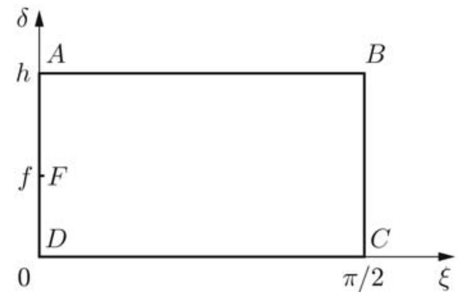


Fig. 2

Fig. 1. Cross section of the interelectrode gap.

Fig. 2. Plane of the parametric variable $t$.

Introducing the characteristic current density $i_0$, the characteristic length $H$, and the dimensionless variables

$$i_0 = \frac{\rho V_c}{\varepsilon}, \quad H = \varkappa \frac{u_a - u_c}{i_0}, \quad x = \frac{x_1}{H}, \quad y = \frac{y_1}{H}, \quad n = \frac{n_a}{H},$$

we represent the complex potential in dimensionless form as

$$W(z) = \varphi(x, y) + i\psi(x, y), \quad z = x + iy, \qquad W(z) = \frac{f(z) - iu_c}{u_a - u_c}.$$

The function $\psi$in the interelectrode gap satisfies the Laplace equation with the boundary conditions on the electrodes

$$\psi_a = 1, \qquad \psi_c = 0; \tag{1}$$

$$\Big(\frac{\partial \psi}{\partial n}\Big)_a = a + b\cos\theta, \qquad a = -\frac{a_1}{a_0 i_0}, \qquad b = \frac{1}{a_0}. \tag{2}$$

In the hydrodynamic interpretation of the electric field model, condition (2) defines the hodograph of the velocity $V$ of the fictitious flow of an ideal incompressible fluid on the anode boundary:

$$V = a + b\cos\theta \tag{3}$$

($\theta$ is the argument of the velocity vector).

**Formulation of the Problem and Numerical-Analytical Solution.** A cross section of the interelectrode gap is presented in Fig. 1. Because of the symmetry of the interelectrode gap, we consider only its left side. The line $CD$ is the boundary of the cathode, and the symmetry lines $BC$ and $DF$ are the current lines orthogonal to the equipotential lines of the electric field. The vector $\boldsymbol{V}_c$ specifies the direction of motion of the cathode. The angles between the tangents to the arc $CD$ at the points $C$ and $D$ and the abscissa are equal to zero and $\pi$, respectively. We divide the required anode boundary into two regions. In the region $AB$, metal dissolution occurs according to condition (2). In the region modeled by the vertical rectilinear region $AF$, metal dissolution does not occur. In this region, the current density changes from $i_{\rm cr}$ at the point $A$ to zero at the point $F$.

A hydrodynamic analog of the problem in question is the problem of the evolution of the boundary $AB$ with a specified variation in the velocity (3) in the theory of plane steady-state flow of an ideal incompressible fluid.

To solve the problem, we introduce an auxiliary complex variable $t = \xi + i\delta$ which varies in the region $D_t$ ($0 \leqslant \xi \leqslant \pi/2$ and $0 \leqslant \delta \leqslant h$), where $h = \pi|\tau|/4$, $\tau = i|\tau|$ (Fig. 2) and determine a function $z(t)$ which conformably maps the rectangle $D_t$ onto the flow region. In this case, the correspondence of the points shown Figs. 1 and 2 is required.

According to conditions (1), the complex potential $W(t) = \varphi(t) + i\psi(t)$ satisfies the boundary conditions

$$\psi(t) = \begin{cases} 0, & t = \xi, \quad \xi \in [0, \pi/2], \\ 1, & t = i\delta, \ \delta \in [f, h], \quad t = \xi + ih, \ \xi \in [0, \pi/2]. \end{cases}$$

On the symmetry lines $DF$ and $BC$, the function $\varphi(t)$ takes a constant value. Without loss of generality, we assume that

$$\varphi(t) = \begin{cases} 0, & t = i\delta, \quad \delta \in [0, f], \\ \varphi_0, & t = \pi/2 + i\delta, \quad \delta \in [0, h]. \end{cases}$$

Using the method of conformal mapping, we find the derivative of the complex potential and the parameter $\varphi_0$:

$$\frac{dW}{dt} = N_1 F_1(t), \qquad N_1 = \Big( \int\limits_0^f F_1(ix)\, dx \Big)^{-1}, \qquad \varphi_0 = N_1 \int\limits_0^{\pi/2} F_1(x)\, dx,$$

$$F_1(t) = \Big( \frac{\vartheta_3(2t)\vartheta_3(0) - \vartheta_2(2t)\vartheta_2(0)}{\vartheta_2(2fi)\vartheta_3(2t) - \vartheta_3(2fi)\vartheta_2(2t)} \Big)^{1/2},$$

where $\vartheta_i(u)$ ($i = \overline{1,4}$) is a theta function with periods $\pi$ and $\pi\tau$ [9].

We consider the Joukowski function $\chi(t) = \ln(dW/(V_0 dz)) = r - i\theta$, where $r = \ln(V/V_0)$ and $V_0 = a + b$. In the rectilinear regions of the boundary, its imaginary part is a piecewise -constant function. Let a continuous function $\theta(s)$ be specified on the arc $CD$, where $s$ is the length of the arc reckoned from the point $D$ (see Fig. 1). Introducing the curvature $K(\theta)$ of the arc $CD$, we obtain the boundary condition

$$\frac{d\theta}{d\xi} = -\frac{K(\theta)}{V_0} \Big| \frac{dW}{d\xi} \Big| \exp(-r(\xi)), \qquad \xi \in [0, \pi/2].$$

Condition (3) leads to

$$a + b\cos\theta(t) - V_0 \exp(r(t)) = 0, \qquad t = \xi + ih, \quad \xi \in [0, \pi/2], \quad r(\pi/2 + ih) = 0.$$

The function $\chi(t)$ can be represented as the sum $\chi(t) = \chi_*(t) + \Omega_1(t) + \Omega_2(t)$, where $\Omega_k(t) = \nu_k(t) + i\varepsilon_k(t)$ ($k = 1, 2$) are analytical in the range of the variable $t$ of the function. The function $\chi_*(t) = r_*(t) - i\theta_*(t)$, $r_* = \ln(V_*/V_0)$ corresponds to the flow in the case where the arc $DC$ is replaced by the straight-line segment $\theta_*(\xi) = \theta(0)$, $\xi \in [0, \pi/2]$, and on the boundary $AB$, the equality $\operatorname{Re}\chi_*(\xi + ih) = 0$, $\xi \in [0, \pi/2]$ holds. In the range of the variable $t$, the functions $\chi(t)$ and $\chi_*(t)$ have similar singularities.

Using the Chaplygin method of singular points [4], we obtain

$$\chi_*(t) = \frac{1}{2} \ln \frac{\vartheta_1(t - if)\vartheta_1(t + if)}{\vartheta_4(t - if)\vartheta_4(t + if)} - \pi i.$$

Let us require that the unknown functions $\Omega_k(t)$ satisfy the boundary conditions

$$\begin{aligned} &\varepsilon_1(t) = \varepsilon_2(t) = 0, && t = i\delta, \quad \delta \in [0, h], \\ &\varepsilon_2(t) = 0, && t = \xi, \quad \xi \in [0, \pi/2], \\ &\varepsilon_1(t) = \pi, \quad \varepsilon_2(t) = 0, && t = \pi/2 + i\delta, \quad \delta \in [0, h], \\ &\nu_1(t) = 0, && t = \xi + ih, \quad \xi \in [0, \pi/2]. \end{aligned} \tag{4}$$

From a comparison of the boundary conditions for the functions $\chi(t)$ and $\chi_*(t)$, it follows that these functions should satisfy the conditions

$$\frac{d\varepsilon_1}{d\xi} = \frac{K(\theta(\xi))}{V_0} \rho(\xi) \exp[-(\nu_1(\xi) + \nu_2(\xi))], \qquad \xi \in [0, \pi/2],$$

$$\rho(\xi) = \Big| \frac{dW}{d\xi} \Big| \exp(-r_*(\xi)) = N_1 F_2(\xi), \qquad F_2(\xi) = F_1(\xi) \sqrt{\frac{\vartheta_4(\xi - if)\vartheta_4(\xi + if)}{\vartheta_1(\xi - if)\vartheta_1(\xi + if)}}; \tag{5}$$

$$a + b\cos(\theta(t)) - V_0 \exp(\nu_2(t)) = 0, \qquad \nu_2(\pi/2 + ih) = 0, \tag{6}$$

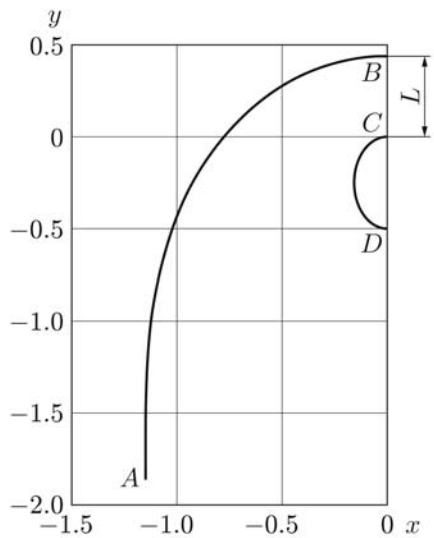


Fig. 3. Calculated cathode and anode boundaries.

where $\theta(t) = \theta_*(t) - \varepsilon_1(t) - \varepsilon_2(t)$, $t = \xi + ih$, and $\xi \in [0, \pi/2]$. Integrating expression (5) with respect to the variable $\xi$ in the segment $[0, \pi/2]$, we obtain

$$\frac{1}{V_0} \int\limits_0^{\pi/2} K(\theta(\xi))\rho(\xi) \exp\left[-(\nu_1(\xi) + \nu_2(\xi))\right] d\xi = \pi. \tag{7}$$

Differentiating relation (6) with respect to the variable $\xi$, we obtain

$$\theta'(\xi + ih) = 0 \quad \text{at} \quad \xi = 0. \tag{8}$$

This condition coincides with the Brillouin–Will smooth separation condition which is well known in hydrodynamics [5, 6].

By virtue of conditions (4), the functions $\Omega_k(t)$ $(k = 1, 2)$ can be expanded in series with real coefficients:

$$\Omega_1(t) = 2(h + it) + 2\sum_{n=1}^{\infty} c_n \sinh\,(2(h + it)n),$$

$$\Omega_2(t) = b_0 + \sum_{n=1}^{\infty} b_n \cos(2tn), \qquad b_0 = \sum_{n=1}^{\infty} (-1)^{n+1} b_n \cosh\,(2hn). \tag{9}$$

The coefficients of series (9) and the parameters $|\tau|$ and $f$ are determined from Eqs. (5)–(8). All the required geometrical flow characteristics can be found using the parametric dependence

$$\frac{dz(t)}{dt} = \frac{\exp(-\chi(t))}{V_0} \frac{dW}{dt}. \tag{10}$$

As an example, we consider the case where the cathode boundary is an ellipse whose foci are located on the ordinate. The curvature of the ellipse is determined from the formula

$$K(\theta) = (1 - \varepsilon^2 \sin^2\theta)^{3/2}/p, \qquad p = a_2^2/b_2, \qquad \varepsilon = \sqrt{b_2^2 - a_2^2}/b_2,$$

where $a_2$ and $b_2$ are the half-axes of the ellipse.

To solve the problem, we specify the values of the half-axes of the ellipse $a_2$ and $b_2$, the coefficients $a_0$ and $a_1$ which characterize the properties of the electrolyte, and the characteristic flow density $i_0$. The problem is solved numerically using the collocation method. The system of equations for calculating the coefficients of series

(9) is solved using the Newton method, together with Eqs. (7) and (8), from which the parameters $|\tau|$ and $f$ are determined. Then, the geometry of the anode boundary is determined by using the parametric dependence (10).

The calculations were performed for the following parameter values: $a_2 = 0.15$, $b_2 = 0.25$, $a_0 = 0.906$, $a_1 = -12.818$, and $i_0 = 50$ A/cm$^2$. The calculations of the positions of the cathode boundary and anode boundary are presented in Fig. 3. The gap size in the section $BC$ is equal to $L = 0.434$, and the coordinates of the point $A$ are $x = -1.144$ and $y = -1.860$.

**Conclusions.** Condition (8) was obtained which can be called the smooth-separation condition, by hydrodynamic analogy. This condition allows one to determine the single possible shape of the anode boundary that satisfies boundary condition (2).

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